# DOBIN - DIMENSION REDUCTION FOR OUTLIER DETECTION 

JOINT WORK WITH ROB HYNDMAN DOB IN : TO INFORM AGAINST, SPECIALLY TO THE POLICE


Meta-learning study on outlier detection


# BUT, NOT MUCH FOCUS ON DIMENSION REDUCTION METHODS FOR OUTLIER DETECTION 

## HIGH DIMENSIONAL OUTLIERS MAY NOT BE OUTLIERS IN LOW DIMENSIONAL PROJECTIONS!

## DOBIN

- Distance-based Outlier Basls using Neighbours
- dob in : To inform against, specially to the police
- Finds a set of basis vectors tailored for outlier detection
- First basis vector - in the direction of highest outlyingness
- Second basis vector - in the next highest direction of outlyingness
- And so on ...


## THE $Y$ SPACE

- For each point $x$ in the original space, find $k$ nearest neighbours $z_{1}, z_{2}, \ldots, z_{k}$
- $x=\left(x_{1}, x_{2}, \ldots, x_{n}\right)$ and $z_{i}=\left(z_{i 1}, z_{i 2}, \ldots z_{i n}\right)$
- $y=\left[\left(x_{1}-z_{i 1}\right)^{2}, \ldots,\left(x_{n}-z_{i n}\right)^{2}\right]$
- $y=\left(x-z_{1}\right)^{E 2}, E 2$ denotes element wise squares
- So, the $Y$ space is an inter-distance space
- If $x \in R^{n}$, then $y \in R^{n+}$
- Remove points with small inter-distances, $y_{1}+y_{2}+\cdots+y_{l}<M$ for some $M$


## EXAMPLE: Y SPACE



## COMPARISON

## $X$ SPACE

- A point is an input observation
- Distance is not a linear combination of $x$ coordinates


## Y SPACE

- A point gives the inter-distancevector between two neighbouring points in the $X$ space
- Distance in $X$ is a linear combination of $y$ coordinates

DISTANCES

- Distance between two points $\operatorname{dist}\left(x_{i}, x_{j}\right)^{2}=\left(x_{i}-x_{j}\right)^{T}\left(x_{i}-x_{j}\right)$
- More generally $\operatorname{dist}\left(x_{i}, x_{j}\right)^{2}=\left(x_{i}-x_{j}\right)^{T} S\left(x_{i}-x_{j}\right)$, where $S$ is a symmetric, positive definite matrix
- $\operatorname{dist}\left(x_{i}, x_{j}\right)^{2}=\left\langle\eta,\left(x_{i}-x_{j}\right)^{E 2}\right\rangle$, element wise squares
- Using $Y$ space
- $\operatorname{dist}\left(x_{i}, x_{j}\right)^{2}=\left\langle\eta, y_{l}\right\rangle$
- $\sum \operatorname{dist}\left(x_{i}, x_{j}\right)^{2}=\sum\left\langle\eta, y_{l}\right\rangle$


## MAXIMISING DISTANCE

- Want to maximise sum of distances between points
- max $\sum \operatorname{dist}\left(x_{i}, x_{j}\right)^{2}$
- Our problem: Find $\eta$ such that

$$
\begin{array}{ll}
\max \sum\left\langle\eta, y_{l}\right\rangle \\
\text { Subject to } & \|\eta\|=1
\end{array}
$$

- Because dist $\left(x_{i}, x_{j}\right)^{2}=\left\langle\eta, y_{l}\right\rangle$
$X$ Space

$\max \sum\left\langle\eta, y_{l}\right\rangle=$ sum of all projections in $\eta$ direction



## GEOMETRICALLY

Y Space


## SOLVING IT

- Using Lagrange multipliers

$$
\eta=\frac{\sum y_{l}}{\left\|\sum y_{l}\right\|}
$$

# WE FOUND THE DIRECTION THAT MAXIMIZES DISTANCES BETWEEN POINTS. 

BUT HOW DO WE GET A BASIS?

## VECTORS PERPENDICULAR TO $\eta$

$$
\begin{aligned}
& \text { Let } \begin{aligned}
\eta_{1} & =\eta \\
y & =y_{p}+y_{\perp} \\
\text { But } y_{p} & =\left\langle y, \eta_{1}\right\rangle \eta_{1} \\
y_{\perp} & =y-\left\langle y, \eta_{1}\right\rangle \eta_{1}
\end{aligned}
\end{aligned}
$$

All $y_{\perp}$ are perpendicular to $\eta_{1}$

## SECOND VECTOR $\eta_{2}$

All $y_{\perp}$ are perpendicular to $\eta_{1}$

$$
\eta_{2}=\frac{\sum y_{\perp}}{\left\|\sum y_{\perp}\right\|}
$$

So $\eta_{2}$ is perpendicular to $\eta_{1}$

## A BASIS

- Continue this way

$$
\begin{gathered}
y_{\perp}=y-\left\langle y, \eta_{1}\right\rangle \eta_{1}-\left\langle y, \eta_{2}\right\rangle \eta_{2} \\
\eta_{3}=\frac{\sum y_{\perp}}{\left\|\sum y_{\perp}\right\|}
\end{gathered}
$$

$$
\text { Basis }\left(\eta_{1}, \eta_{2}, \ldots, \eta_{n}\right)
$$

## HOW DO WE TEST DOBIN?



## O3 PLOTS




## ELECTION2005 DATASET <br> R package mbgraphic

## DIAMONDS DATASET <br> R package ggplot2

O3 plot of outliers found by at least 2 of 3 methods


TIME SERIES DATA - MAD RIVER NEAR SPRINGFIELD, OH
R package tsdl



## LESMIS DATASET

R package SOMbrero


## CLASSICS FROM GUTENBERG

## - 22 classics downloaded from Gutenberg project

- Alice in Wonderland, Anna Karenina, Bleak House, Emma, Frankenstein, Gullivers Travels, Jude the Obscure, Lord Jim, Mansfield Park, Middlemarch, Moby Dick, Northanger Abbey, Persuasion, Pride and Prejudice, Sense and Sensibility, Silas Marner, Sons and Lovers, The Life and Opinions of Tristram Shandy, Wizard of Oz, Ulysses, Vanity Fair and War and Peace



## CLASSICS FROM GUTENBERG



## USES OF DOBIN

- Dimension reduction for outlier detection
- Visualization of outliers


## THANK YOU!

- R package dobin is on CRAN http://bit.ly/cran dobin
- Slides at https:/ /sevvandi.netlify.com /
- Paper on RG: http://bitly/paper dob in

- @ @sevrandik
- Osevvandi

